Logics

Where are we? In retrospect

(HP2T)

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The World and the Mind

Most (all?) of us think that the world is how we see (perceive) it.

That is, we confuse the world with our mental representation of the world itself.

Is this a correct assumption?

Four types of mental mistakes

- ***Perception* mistakes:** see what is not there and vice versa (e.g. *a face in the moon*, *the face of an old/young woman*).
- ***Conceptualization* mistakes:** same world, with different words for same percept (e.g., *collided*, *bumped*, *hit*, *smashed*) or same words for two percepts (e.g., *which car*? an *automobile* or a *train car*?).
- ***Representation* mistakes:** partial, biased, wrong memories (*people living* vs. *people dying*).
- ***Reasoning* mistakes:** wishful thinking, derivation of wrong conclusions (e.g., all *fallacies*).

These mistakes co-occur in any complex mental activity

Mistakes or different mental contructions?

- *Perception mistakes or different percepts?*
- *Conceptualization mistakes or different conceptualizations/ concepts?*
- *Representation mistakes or different mental representations?*
- *Reasoning mistakes or different reasoning processes?*

Logic

- ***Logic*** gives us with a structured framework for *modeling* the world, and our way of thinking, that is *reasoning***,** about it.
- ***Logic*** is crucial for the (minimization of the impact of) *modeling* differences in *perception*, *conceptualization*, *representation*.
- ***Logic*** is crucial for *automation of reasoning*
- ***Logic*** is a key enabler in Computer Science and Artificial Intelligence!
- ***We*** will formalize all four mental activities into appropriate world modeling logics, ***we*** will show how to compose them, ***we*** will use them for the automation of reasoning in machines.

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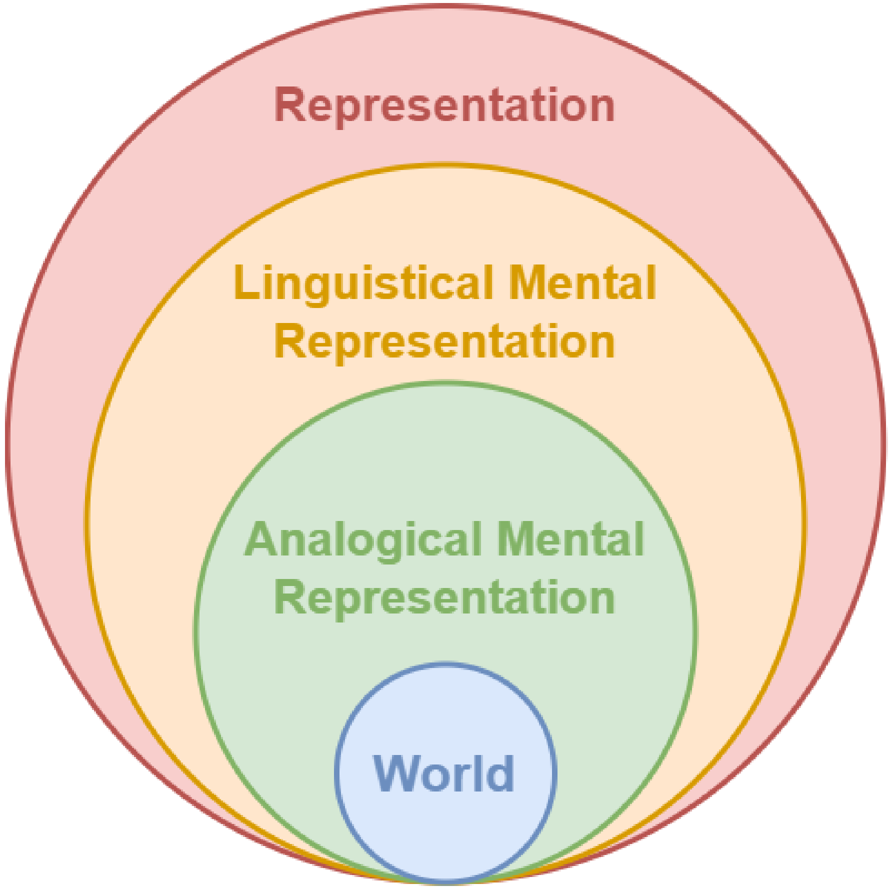
Representations (continued)

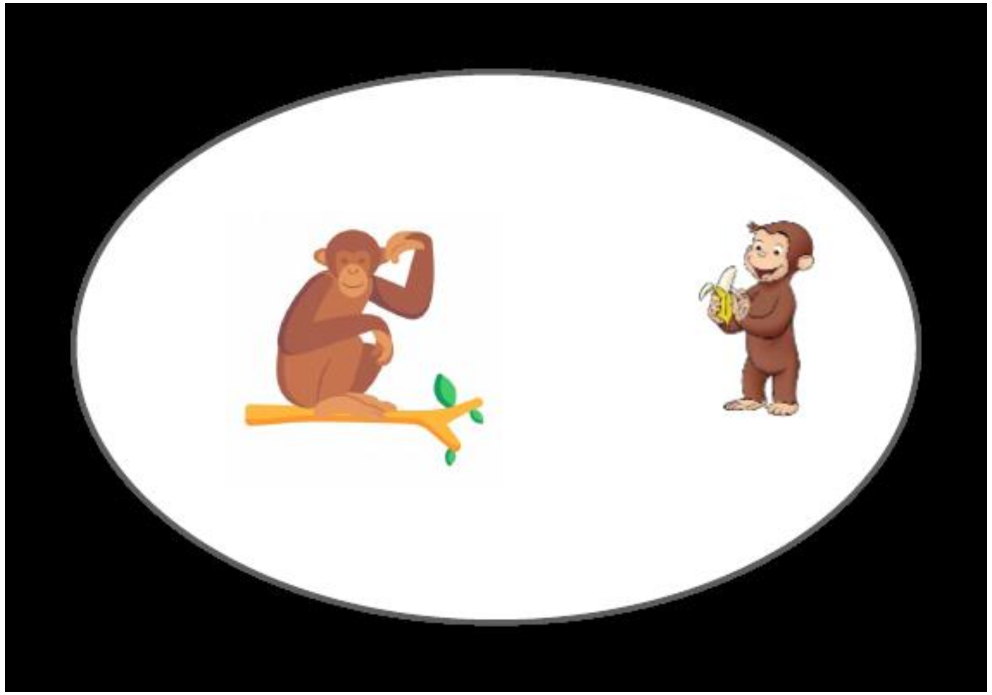


Two types of representation

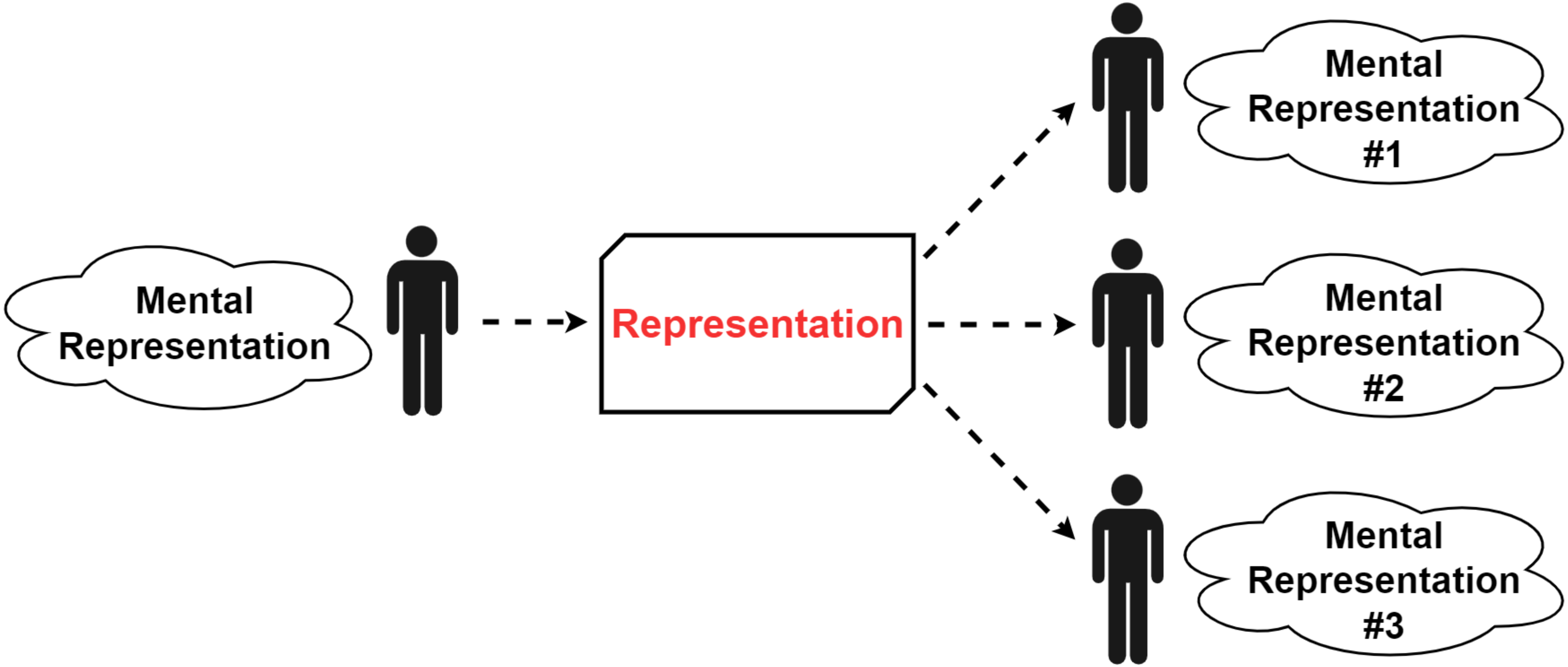
- **Analogical representations**
- **Linguistic representations**

Linguistic vs. analogical representations

- There is a tree
- There is a banana
- The monkey is eating a banana
- The monkey is sitting on a tree
- The monkey is scratching its head



Mental representations of representations



Mental representations of representations (continued)

Observation (Difficulty). The previous slide may suggest that there is no solution to the problem of subjectivity of mental representations. However this is not the case!

Observation (Requirement on representations). Representations are built with the goal of minimizing the probability of different interpretations and, therefore, of mental representations.

Observation (Using representations). Different interpretations may still arise. Risk minimized (not eliminated) via software and knowledge engineering methodologies.

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World Model

Definition (World model). Given a **Domain of interpretation** D, a **world model** W is defined as

$$W = \langle L_a, D, I_a \rangle$$

where L_a is an **assertional language**, $I_a : L_a \rightarrow D$ is an **interpretation function** and L_a is **correct** with respect to D.

Observation (World model). L_a is not necessarily complete.

World models, theories and models

Definition (Theory, model). Given a world model

$$W = \langle L_a, D, I_a \rangle$$

then, given M and T_a defined as follows,

$$M = \{f\} \subseteq D$$
$$T_a = \{a\} \subseteq L_a$$

M and T_a are, respectively, a **model** of T_a and a **theory** of M, if **T_a is correct for M.**

Observation (Theory). Theories are not necessarily complete.

World representations

Definition (World representation). Given a world model

$$W = \langle L_a, D, I_a \rangle$$

then

$$R = \langle T_a, M \rangle$$

is a **world representation**, with

$$M = \{f\} \subseteq D$$
$$T_a = \{a\} \subseteq L_a$$

where M and T_a are, respectively, a **model** of T_a and a **theory** of M in W.

Definition (Canonical world representation). A world representation is **canonical** when M is the **canonical model** of T_a (that is, T_a is **maximal** for M).

Example world models

- **Natural language, domain, application lexicons:** a semi-formal world model of vocabularies
- **EER Models:** a semi-formal world model of knowledge about the world
- **ER models:** a semi-formal world model of knowledge about the world
- **Relational Data Bases:** a semi-formal world model of the entities of the world
- **Knowledge Graphs:** a semi-formal world **model** of the world in its entirety
- **Natural languages:** an informal world model of the world in its entirety
- **Ontologies, language teleontologies:** a formal **world model of vocabularies**
- **Knowledge teleontologies:** a formal world model of knowledge about the world
- **Teleologies /etype Graphs (ETGs):** a formal world model of knowledge about the world
- **Entity Graphs (EGs):** a formal world model of the entities of the world

... but there are many more, for instance modeling change /time (e.g., UML diagrams, FSTs)

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What world models do / do not represent

World models …

- Allow us to formalize linguistic and analogic knowledge about the world and how they are connected
- Allow us to distinguish correct and complete model descriptions from their counterparts;
- Do not provide us the means for deciding what is true and what is false in the world;
- Do not allow to reason about what we know.

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World logics - intuition

Intuition (World logics): Based on the representational choices made by world models and world representations, world logics allow for the most basic form of reasoning about world representations. They have three main components, that is:

- The intended **model**
- A **set of assertions** defining an input theory describing the intended model
- A **world entailment relation** which allows to decide whether the input assertional theory is actually a theory of the intended model (what is **True** and what is **False** in the intended reference model).

A **world logic** is any representation which encodes the three types of information described above.

Observation (World entailment, reasoning). Once, based on a selected world model, one has constructed a world representation, the next issue is to use it to **reason** about it, that is to reach conclusions about it. Building upon the notion of **correctness** provided by the interpretation function, world entailment provides a mechanism, an algorithm, for deciding whether an input set of assertions is actually a theory if the intended model.

Observation (World entailment, entailment). As discussed later, there are complex forms of entailment, modeling, complex, **language driven** forms of reasoning. All of them, however, ultimately use world entailment as the key operation, as from its name, for deciding whether something is actually true in the world. Language driven reasoning can elaborate upon but cannot substitute checking truth in the world (that is, in the intended model).

World entailment

Definition (World entailment) Let W =< L_a, D, I_a > be a world model. Let L_a be an assertional language. Let M={f} ⊆ D be the intended model. Let $T_a \subseteq L_a$ be an assertional theory. Then $|=_{La}$ is an **world entailment relation** that **associates facts in M with assertions in T_a**, in formulas

$$|=_{La} \subseteq M \times T_a$$

We also write

$$M \ |=_{La} T_a$$

and say that M (**world**) **entails** T_a. We write M $|=$ T_a instead of M $|=_{La}$ T_a when no confusion arises.

World Entailment

Proposition (World entailment). Let W =< L_a, D, I_a > be a world model. Let M ⊆ D. Let $T_a \subseteq L_a$ ($a \in L_a$) be an input assertional theory (assertion). Then

M|= T_a if, for all $a \in T_a$, a is **True** in M

M|= $\{a\}$, written M|= a, if a is **True** in M

Observation (World entailment). T_a is entailed by M if all its assertions are true in M. World model entailment reduces entailment to checking, via the interpretation function, for truth / falsity in the model.

World logics and world logic representations

Definition (World logic). Given a **world model** $W = \langle L_a, D, I_a \rangle$, a **world logic** L_W for W is defined as

$$L_W = \langle W, |=_{La} \rangle$$

where $|=_{La}$ is a **world entailment relation**.

Definition (World logic representation). Given a world logic $L_W = \langle W, |=_{La} \rangle$, a (**world logic**) **representation** is defined as

$$R = \langle T_a, M \rangle$$

with

$$M = \{f\} \subseteq D$$
$$T_a = \{a\} \subseteq L_a$$

where M and T_a are, respectively, a **model** of T_a and a **theory** of M in L_W.

Observation (World logic representation). A world logic representation is the same as that of its world model.

Example world logics

The world logics we have studied:

- **LoE**: the Logic of Entities. It is a base world logic. It formalizes (only) the **interpretation function based** entailment of **world models**;
- **LoD**: the Logic of descriptions. It formalizes how the LoE language can be extended to allow for **commonsense language definitions** and **commonsense knowledge descriptions**, and reasoning about them;
- **LoDe**: The Logic of Entity Bases. It exploits LoD, that is, commonsense knowledge and reasoning, to enhance **world model entailment**, as formalized in LoE up to **commonsense world entailment**.

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LoDE hardly allows for Disjointness

Observation (LoDE hardly allows for disjointness). LoDE allows to say that two assertions are disjoint.

Example (LoDE hardly allows for disjointness). Consider the assertions "my T-shirt is green" and "my T-shirt is grey". Their relation can be formalized in LoDE as

$$\text{Color(myTshirt, green)} \perp \text{Color(myTshirt, grey)}$$

as a consequence, via expansion, of the general LoD description

$$\text{Color(object, green)} \perp \text{Color(object, grey)}$$

applied to the LoE assertion

$$\text{object(myTshirt)}$$

Observation (LoDE hardly allows for disjointness). There is no obvious way to represent disjointness (graphically) in EGs beyond lexicon elements. You need to have language assertions.

LoDE does not allow for Negative knowledge

Observation (LoDE does not allow for negative knowledge). LoDE does not allow to say that the set denoted by an assertion is (exactly) the complement (with respect to the Universe of Interpretation) of another assertion.

Example (LoDE does not allow for negative knowledge). Consider the assertions "the door is open" and "the door is closed". They can be formalized as the LoDE assertion

⊥ ≡ state(door #1, open) ⊓ state(door #1, closed) [*same as* state(door #1, open) ⊥ state(door #1, closed)]

As a consequence, via expansion, of the general LoD description

state(door, open) ⊥ state(door, closed)

applied to the LoE assertion

door(door#1) ⊥ door(door#1)

However, we also have the following fact

T ≡ state(door, open) ⊔ state(door, closed) [*same as* state(door#1, open) ≡ ¬ state(door#1, closed)]

which is NOT a LoDE assertion (negation is not allowed in LoDE).

Observation (LoDE does not allow for negative knowledge). In case of negation of primitive etypes, one can introduce two new words "stateOpen" and "stateClosed" refining the meaning of "state". But this grows the complexity of the EG and it may result in an increase of cost in terms of reasoning efficiency. Plus, it can be argued that **perceiving** a "open door" is different from perceiving a "closed door" and **deducing** that, therefore, it is not closed (what is done in Language logics).

LoDE does not allow for Partial knowledge

Observation (LoDE does not allow for partial knowledge). LoDE allows to state what is the case. But it says nothing about "the rest", namely, what is not mentioned explicitly in the (unfolded and expanded) EG, **because it is unknown**.

Example (LoDE and partial knowledge). Consider the assertion "my T-shirt is green". Consider now the following two assertions:

- What about the assertion "my t-shirt is grey"?
- What about the assertion "my paints are grey"?

Can they be part of the same world description? There is a key distinction between **partial knowledge** and **negative knowledge**, which is not captured by LoDE EGs.

Observation (Closed World Assumption - CWA). CWA is a partial solution to the problem partiality. It does not allow to reason about partiality. It only assumes complete knowledge.

LoDE does not allow for Logical Consequence

Observation (LoDE does not allow for Logical Consequence). LoDE allows to state what is the case. But it says nothing about **what can be deduced by computing consequences**, that is , reasoning, **from what is known.**

Intuition (Logical Consequence). Intuitively, we say that a formula is a logical consequence of another formula if it can be deduced from it by (logical) reasoning.

Example (Logical Consequence). If one knows the following two facts

- Whenever Fausto is not at home his children make a lot of noise
- Fausto's children are quite

then (s)he can conclude that

- Fausto is at home

Observation ("Whenever Fausto is not at home his children make a lot of noise"). This sentence cannot be expressed in LoDE.

What World Logics do / do not represent

World logics …

- Provide us with the means to represent and reason about world models, that is, to decide what is true and what is false in world model, represented as entity graphs (EGs);
- Provide us with the means to represent and reason about the commonsense meaning of language (words);
- Provide us with the means to represent and reason about commonsense knowledge (descriptions);
- Do not allow us to represent and to reason about what is not depicted in the reference world model, but that everybody knows (being commonsense knowledge) or can be deduced via (commonsense) reasoning.

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Logics – Intuition (reprise)

Observation (World logics). World logics formalize how truth in a model can be reasoned about in a (logical) theory (that is, a linguistic representation of the world). They are the key basic element, via the entailment relation, for the formalization of (**logical) reasoning.**

Observation (Language logics). Logical reasoning is linguistic reasoning, that is, reasoning in a predefined language. Logical reasoning is implemented using **(language) logics** which allow **to draw conclusions** from the true facts computed by world logics.

Observation (World and language logics). Language logics use world logics as **oracles** which provide information about what is true/ false in the intended model. Language logics implement reasoning.

Assertions (reprise)

Definition (Assertion). An **Assertion** a is an **atomic sentence**, that is, a sentence which cannot be decomposed into simpler sentences, which unambiguously describes a single **fact**. For any assertion a we have

$$a \in L_a = \{a\}$$

Observation (From facts to assertions) The mapping from facts to assertions is in full control of the modeler. We have the following:

- For formation rules there is no specific recipe. The general idea is that they should generate a syntax which makes it as easy as possible for people to understand the underlying facts (see above);
- A special case, see above, is when there is no need of formation rules.

Types of assertions (reprise)

Definition (Assertion). An **assertion** a has one of the following five forms

- Assertion starting fact: $u_i \in C_j$,
- Assertion starting fact: *Tuple of Units memberOf relation:* $< u_1, ..., u_n > \in R^n$,
- Assertion starting fact: *Class subsetOf Class:* $C_i \subseteq C_j$,
- Assertion starting fact: *Relation subsetOf relation:* $R_i^n \subseteq R_j^n$
- Assertion starting fact: *Relation subsetOf tuple of classes and viceversa:*
  - $R^n \subseteq C_1 \times ... \times C_n$
  - $C_1 \times ... \times C_n \subseteq R^n$

Different assertional languages differ in the choice of facts to be asserted. See examples below.

Judgements

Intuition (Judgement by Merriam-Webster on line). A **judgement** is the process of forming an opinion or evaluation by discerning and comparing.

Intuition (Economics Dictionary of Arguments on line). A **judgment** differs from a statement (an "assertion" in our terms) in that it also asserts the truth of its content.

Examples (Judgement):

- It is true that the door is open
- It is false that the door is open

… as different from the assertions

- The door is open
- The door is closed

Observation (Assertion and judgement). An assertion describes (falsely or truly) what is the case. A judgement is a (meta)statement about the truth of an (object) assertion. We have

- A judgement of falsity / truth about a false assertion
- A judgement of truth / falsity bout a true assertion

Propositions

Notion (Aristotle). A **proposition** is a sentence which affirms or denies a predicate of a subject.

Intuition (Proposition). A proposition is an assertion that expresses a judgment about an assertion. That is, we have two types of propositions:

- A certain assertion is False
- A certain assertion is True

Intuition (Value of propositions). A proposition can denote true or false. This is the intended model. A proposition

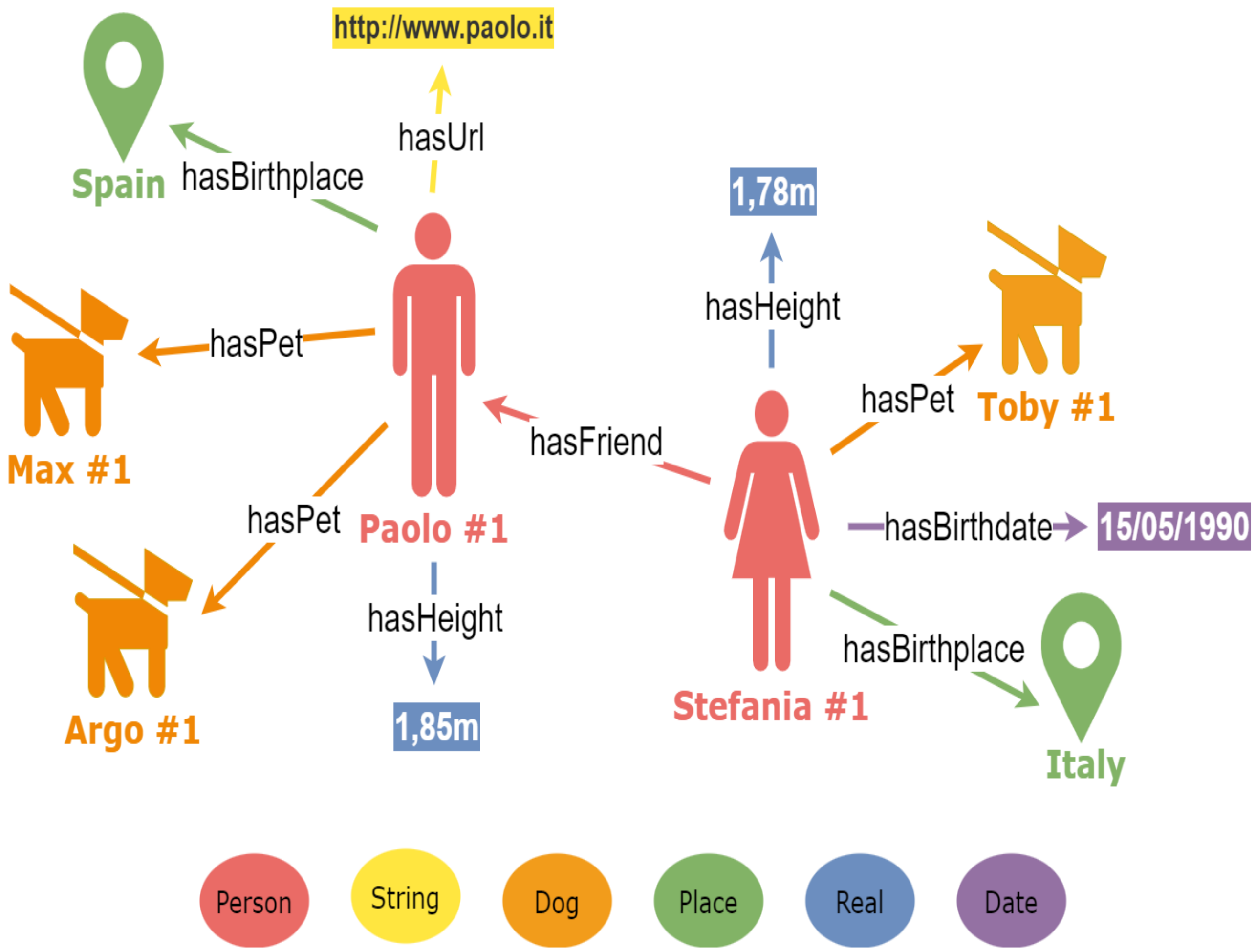
- can be either true or false,
- it must be one or the other,
- and it cannot be both.

Types of propositions

Intuition (Atomic and complex propositions). There are two types of propositions:

- **Atomic propositions**, which are not further decomposable and which are either true or false, and
- **Complex propositions**, composed of atomic propositions, whose truth value depends on the truth value of the atomic propositions of which they are composed.

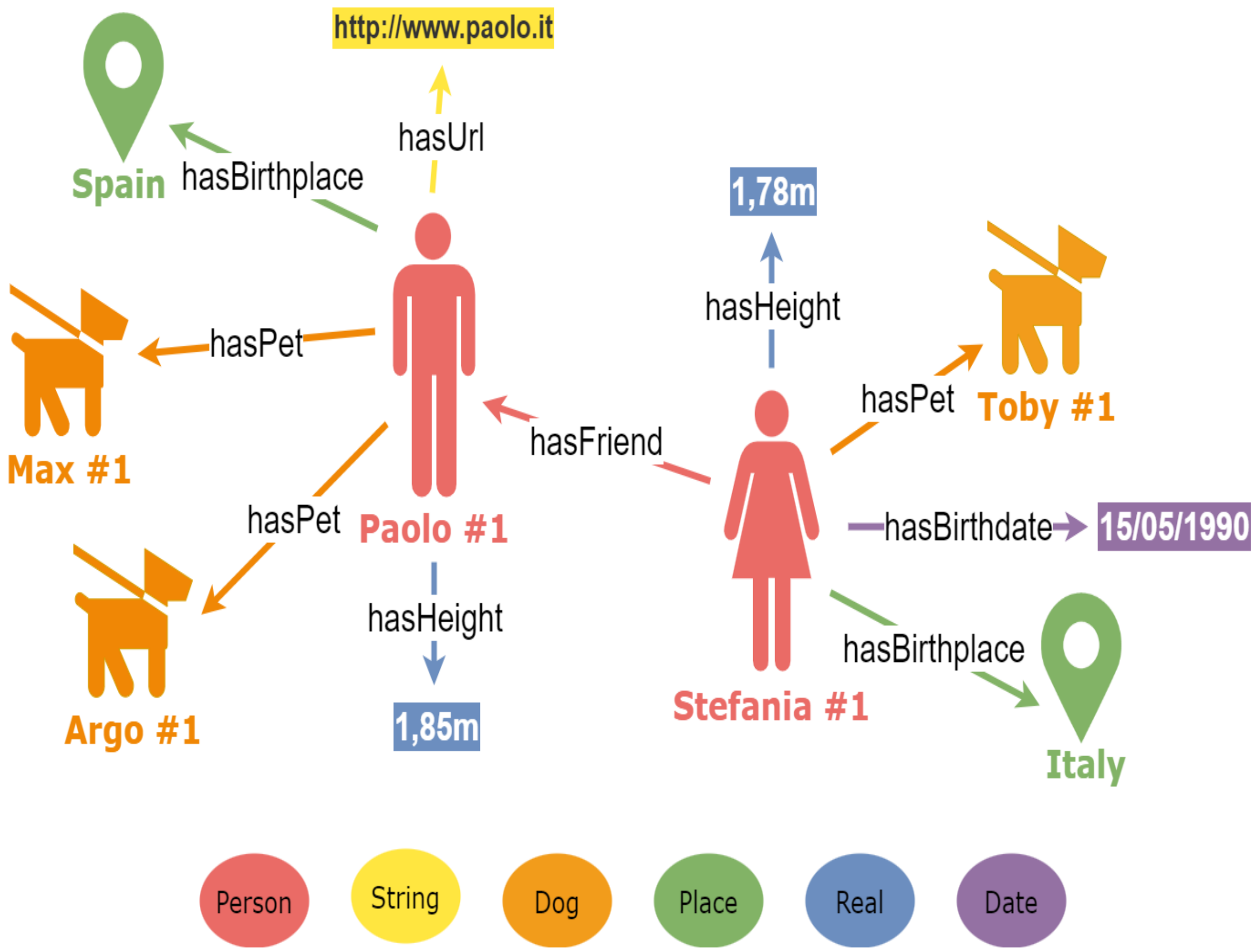
Atomic propositions – A LoDE EG example



Which of the following propositions are intuitively true?

- *HasFriend(Stefania#1,Paolo#1) = T*
- *HasFriend(Stefania#1,Paolo#1) = F*
- *Hasheight(Stefania#1, 2m) = T*
- *Hasheight(Stefania#1, 2m) = F*

Complex propositions – A LoDE EG example



Which of the following complex propositions are intuitively true?

- *HasFriend(Stefania#1,Paolo#1) = F* <u>and</u> *HasHeight(Stefania#1, 2m) = F*
- *HasFriend(Stefania#1,Paolo#1) = F* <u>or</u> *HasHeight(Stefania#1, 2m) =T*

No need of the LoDE EG when computing the truth value of complex propositions! We could have written the two propositions above as:

- F <u>and</u> B
- A <u>or</u> B

with A= F, B= T.

Complex propositions – A language-only example

Example (Complex propositions). Propositions composed out of assertions are in ”quotes”. Within quotes, the judgement is underlined. Natural language connectives are underlined. and, or, not, if ... then, iff, and not have their intuitive meaning.

A = ”It is true that Stefania#1 is a friend of Paolo#1” ;
B = ”It is false that Stefania#1 is a friend of Paolo#1” ;
C = ”It is true that today it rains”;
D = ”It is false that Paolo#1 is a woman”;
E = ”It is true that Paris is the capital of Italy”
G = ”It is false that it is true that Paris is the capital of Italy”

1. A
2. B
3. A and not C
4. A or D
5. (A or D) and E
6. If not E then G
7. A and not A
8. A or not A
9. E iff not G

Exercise. Compute the truth value of the propositions on the right with respect to the LoDE EG.

Truth values of propositions

Intuition (Truth / falsity of atomic propositions). The truth / Falsity of atomic propositions is computed by "asking" a World Logic, the one describing the model described by the assertion whose truth is evaluated. The answer from the World Logic can be **True**, **False**, or **I don't know**.

Intuition (Truth / falsity of complex propositions). The truth / Falsity of complex propositions is computed from the truth value of atomic propositions, based on the entailment relation of the (language) logic. They depend on a proposition to be **True** or **False**.

Observation (truth of atomic vs complex formulas). The truth value of atomic propositions depends on what is the case in the underlying world model and logics (the language they use to describe the world model). The truth value of complex propositions does not depend on the world model. It only depends on the truth valure of atomic propositions, independently of what the underlying assertions assert.

Observation (Reasoning). Reasoning is **linguistic** and independent of what is the case in the world. It only depends on how truth values compose in a statement. In this sense, it is **universal**, meaning by this, **independent of the world as we perceive it** (in our mental analogical representation).

Terminology – formulas (reprise)

Terminology (Formula, well-formed formula, wff). All logics rely on a language, defined in terms of a type 2 Chomsky grammar, composed of an alphabet and a set of formation rules, an interpretation function and a domain of interpretation.

Languages can be distinguished in terms of the objects which constitute their domain of interpretation. We call **formula**, or **well-formed formula**, or **wff** any element of given language which is correctly formed, starting from the alphabet and using the formation rules, independently of their domain of interpretation. Notationally, we write

L = {*w*} to mean that the language L is a set of wffs *w,*

Terminology (Assertion). Assertions, distinct in atomic and complex assertions, are formulas which describe facts of composition of facts as they occur in the intended model. Assertions are used in world logics. Notationally, we write

L = {*a*} to mean that the language L is a set of assertions *a*

Terminology (Proposition). Propositions, distinct in atomic and complex propositions, are formulas which describe what is true in the intended model. Propositions are used in (language) logics. Notationally, we write

L = {*p*} to mean that the language L is a set of propositions *p*

Observation (formula). The distinction among the different types of formulas (e.g., assertion, vs. proposition) is based on what they denote, and it is independent of the specific alphabet and formation rules.

Logics represent positive and negative knowedge

Observation (Representing LoDE positive and negative knowledge). The (explicit) positive and the (implicit) negative knowledge of a LoDE representation is encoded by:

- representing positive knowledge (a true assertion) with the proposition which encodes it (that is: "it is true that …")
- representing negative knowledge (a false assertion) with a proposition which encodes it (that is: "it is false that …")
- requesting that any atomic proposition "P" to take the value True or False, but not both;
- requesting that the domain D allows, for any assertion, to assert that this assertion is either True or False.

Observation (Contents of a domain). A domain D needs only to contain two percepts, that is, the pair <True, False>

Observation (Symbol for negation). The above approach requires that there are 2*N atomic propositions, where N is the number of assertions. The number of atomic propositions can be reduce to N but allowing the symbol not and the complex proposition where "not P", where "P" is an atomic proposition.

Logics represent partial knowledge

Observation (Representing and reasoning about partial knowledge). Language logics model partiality by requesting that models are complete, that is, that they assign a truth value to each atomic proposition allowed by a LoP language L.

Observation (Theory partiality). Theories are allowed to be partial (as in world logics).

Example (LoP complex propositions). Consider the previous example.

A = "It is true that Stefania#1 is a friend of Paolo#1" ;
B = "It is false that Stefania#1 is a friend of Paolo#1" ;
C = "It is true that today it rains";

1. A
2. B
3. A and not C

The theory containing the three propositions on the right will have 2 out of 8 possible models. All of them contain all truths defined by the input theory.

Observation (Contents of a model). A model M has always as many elements, that is truth values as there are atomic propositions in the language.

Logics represent partial knowedge (continued)

Observation (Reference model partiality). As discussed earlier on, reference models are most often incomplete and, as such, they are usually defined by a theory which is incomplete. This is allowed by world logics but not in (language) logics. An incomplete reference model, as it was allowed in world logics corresponds here to a set of models which (see above on theory partiality):

- All share the true propositions defined by the (incomplete) reference model
- Cover all possible combinations of truth value assignments to all propositions whose truth is unknown. The number of models doubles for every proposition whose truth value is unknown

Example (On the incompleteness of reference models). Consider the previous example.

A = "It is true that Stefania#1 is a friend of Paolo#1" ;
B = "It is false that Stefania#1 is a friend of Paolo#1" ;
C = "It is true that today it rains";

1. A
2. B
3. A and not C

A theory containing the three propositions on the right will have 2 out of 8 possible models.

Observation (Maximal theory). In a language logic, a theory, to be maximal, must assign a truth value to all (atomic) propositions allowed by the language. This is different from world logics.

Logics represent Logical consequence (entailment)

Definition (Logical entailment) Let M be a model and T_1, $T_2 \subseteq L$ be two theories and $w \in L$ a formula. Then we write

$$T_1 \models_{\{M\}} T_2 \qquad (T_1 \models_{\{M\}} w)$$

and say that T_1 **(logically) entails** T_2 (w) with respect to the **set of models** {M} if

$$\text{for all } M \in \{M\}, \text{ if } M \models T_1 \text{ then } M \models T_2 \ (M \models w)$$

Observation (Logical entailment, reasoning). Logical entailment implements reasoning in the following way: "Consider a set of models. Whatever we derive from some premises must be true in the same sets of models, a subset of {M}, which make the premises true.

Observation (Which set of models {M}). The choice of {M} is arbitrary. It could be all, one model, or any set chosen more or less arbitrarily.

Notation ({M}). We write $\models$ instead of $\models_{\{M\}}$ to mean that {M} is the set of all models.

Logical entailment – observations

Observation (Opinions/ reasoning fallacies / mistakes). The definition of a logical consequences provides a partial explanation of why there are different opinions, reasoning fallacies or mistakes. Being reasoning completely abstracted from the world, any mistake in the computation of the truth value of an atomic proposition in the reference world logics propagates to the entire reasoning path till a check of the truth value of the conclusion in the underlying world model is done (**reality check**).

Example (Reasoning fallacies). Some examples as from the first lecture

- Misconceptions
- Overgeneralization
- mental filtering
- Catastrophism
- Bias

Logical entailment - properties

Definition (Reflexivity)

$$w \models w$$

Observation (Reflexivity). Every fact entails itself. Knowledge asserts itself as being knowledge. This is the essence of what knowledge is about. That is, if a formula is true in a representation (the memory of what we have perceived or been told), then I'll answer so any time I am asked.

Observation (Reflexivity). Reflexivity, to hold, requires that one is capable of recognizing two occurrences of the same formula (proposition, assertion, fact) to be the same. That is (as stated earlier on) we need a theory of (dis)similarity.

Logical entailment – properties (cont.2)

Definition (Cut)

If T |= w_1 and Σ ∪ {w_1} |= w_2 then T ∪ Σ |= w_2

Observation (Cut) There are two ways to interpret cut.

The first and most common relates to **efficiency**. That is, reasoning can be made efficient by dropping intermediate irrelevant results.

The second relates to **transitivity**. That is, reasoning con be computed by chaining independent reasoning sessions, something that people do all the time during their everyday life.

Logical entailment – properties (cont.3)

Definition (Compactness)

If T |= w then there is a finite subset $T_0 \subseteq T$ such that T_0 |= w

Observation (Compactness). Compactness says that we do not need to use all we know to derive our goal.

This applies also in the case that what we know is infinite (which happen with infinite sets, for instance, with time and space coordinates and regions).

Logical entailment – properties (cont.4)

Definition (Monotonicity)

$$\text{If } T \mid= w \text{ then } T \cup \Sigma \mid= w$$

Observation (Monotonicity) Monotonicity implements a fundamental and intuitive property of knowledge, for instance of scientific knowledge. If knowledge increases then what can be derived from it via reasoning can only increase.

At most it can stay the same if the new piece of knowledge was implied by what is already known.

Logical entailment – properties (cont.5)

Definition (NonMonotonicity)

$$T \models w \text{ and } T \cup \Sigma \text{ not} \models w$$

Observation (NonMonotonicity) Monotonicity is a property which most often does not hold. This is extensively the case with commonsense reasoning, a topic extensively studied in AI.

How many times getting to know something new has forced us to change our mind? Historical AI example: the belief that all birds fly can be defeated by the fact that penguins are birds and they do not fly.

Historical scientific knowledge example: the discovery that it is the earth rotating around the sun, and not vice versa.

Pratical point of view: the logics used in mathematical reasoning and in formal methods, as applied to, e.g., programming languages, are monotonic, while most logics defined in AI are nonmonotonic. Negation by failure, as implemented in relational DBs is nonmonotonic.

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Logic Entailment

Notion 6 (Entailment) Let W =< L_a, D, I_a > be a world model. L_W = ⟨W, $|=_{La}$⟩ be a world logic for W. Let L = {*w*} be a language, with $L_a \subseteq L$. Let M={f} ⊆ D be a set of facts. Let T ⊆ L be a theory. Then $|=_L$ is an **entailment relation** that associates the facts in M with the elements in T, in formulas

$$|=_L \subseteq M \times T, \text{ also written } M \,|=_L T, \qquad (*)$$

subject to the constraint that for all assertions $a \in L_a$,

$$\text{if } M \,|=_{La}\, a \ \text{ then } M \,|=_L\, a' \qquad (**)$$

We also say that M **entails** T and write M |= T when no confusion arises.

Observation (L_a ⊆ L). Usually the assertions $a \in L_a$ get rewritten, to assertions $a' \in L$, under the guarantee of a one-to-one mapping between the two notations.

Observation (L_a ⊆ L). (**) holds only in one direction because of the partiality of world logic models. This revises a previous definition given that we have restricted ourslelved to complete logic models.

Entailment – observations

Observation 1 (Entailment, reasoning). The definition of entailment is made based on a theory T ⊆ L, where there exists a suitable L_a, with L_a ⊆ L. The key intuition is that of extending a reference assertional language to allow for formulas which are not necessarily assertions and, then, to ask about the truth of these formulas.

Observation 2 (Entailment, reasoning). Entailment formalizes the intuitive notion of reasoning. It links what one asserts as being the case with what is true in the model. There are multiple notions of entailment, formalizing different notions of reasoning, even for the same world model, with wildly different properties, still maintaining the properties listed before.

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Logics and logic representations

Definition (Logic). Given a **world model** $W = \langle L_a, D, I_a \rangle$ and a **world logic** $L_W = \langle W, |=_{La} \rangle$, a **logic** L_L for L_W is defined as

$$L_L = \langle W, |=_L \rangle$$

where $L_a \subseteq L$, and $|=_L$ is an **entailment relation**.

Definition (Logic representation). Given a logic $L_L = \langle W, |=_L \rangle$, a (**logic representation** is defined as)

$$R = \langle T, M \rangle$$

with

$$M \subseteq D$$

$$T = \{w\} \subseteq L$$

where M and T are, respectively, a **model** of T and a **theory** of M in L_L.

Logics

Observation (Logics). We will study the following two logics:

- **LoP**. The **Logic of Propositions**, also called **Propositonal Logic**. It allows to reason about and to draw consequence from **propositions**, that is, from judgements about what is true and what is false;
- **LoI**. The **Logic of Interaction**, also known as the **First order Logic**. This logic allows for the use of variables, **existential** and **universal quantification**. It approximates the expressivity of the language used in natural language interactions (towards LLMs).

Observation (LoI). Because of our focus on Computer Science, we will focus on finite domains.

What Logics do / do not represent

Logics …

- Provide us with the means to represent linguistically and reason about the knowledge encoded in a LoDE representation;
- Provide us with the means to represent and reason about negative knowledge, as implicit in a LoDE representation;
- Provide us with the means to represent and reason about partiality, as implicit in a LoDE representation;
- Provide us with the means to draw consequences from what is known via logical consequence, as implicit in a LoDE representation;

Observation (World versus language logics). Differently from world logics, language logic do not have a graphical representation of the knowledge they encode (!). This is because their models are not analogical but, rather, linguistic.

Observation (World versus language logics). Language logics allow to represent and reason about what we do not directly perceive and that, therefore, cannot be represented in an analogical representation. They allow to represent and reason about what we learn it is the case by experience.

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Key notions

- World representation
- World model
- World logic
- (Language) Logic
- Assertions, judgments, propositions, formulas
- Atomic and complex propositions
- Knowledge about disjointness
- Negative knowledge
- Partial knowledge
- Logical consequence /entailment
- Logic entailment properties
- Reflexivity, cut, compactness, (non) monotonicity

Logics
Where are we? In retrospect
(HP2T)